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Unannounced and unattributed, it arrived in the mailbox this fall: a repurposed postcard with Robert Smithson's earthwork, *Spiral Jetty* (1970), on the front and, on the back, instructions for How to Draw & Count Labyrinths. Dia Art Foundation was crossed out and replaced by Cretan Correspondence School of Topology. The card was addressed to Eden Reinfurt, a 16-year-old high school student in New York City. She later surmised that the sender must be Philip Ording, a topologist, professor, and author in Brooklyn.

The postcard cued a wandering back and forth conversation conducted through the U.S. Postal Service. Eden worked through the problems posed on the first postcard, drawing her answers on a return card which went back in the mail. This one took a while, however, as it was misaddressed (some blame Eden's father for the mistake).

Four more postcards followed in sequence, each developing the ideas a bit further and posing additional problems. Some were answered correctly. What follows is the complete correspondence course in six postcards, each annotated here by either teacher or student.

Cover: Eden Reinfurt finding her way out of a painted labyrinth near Houston Street and University Place in downtown New York City

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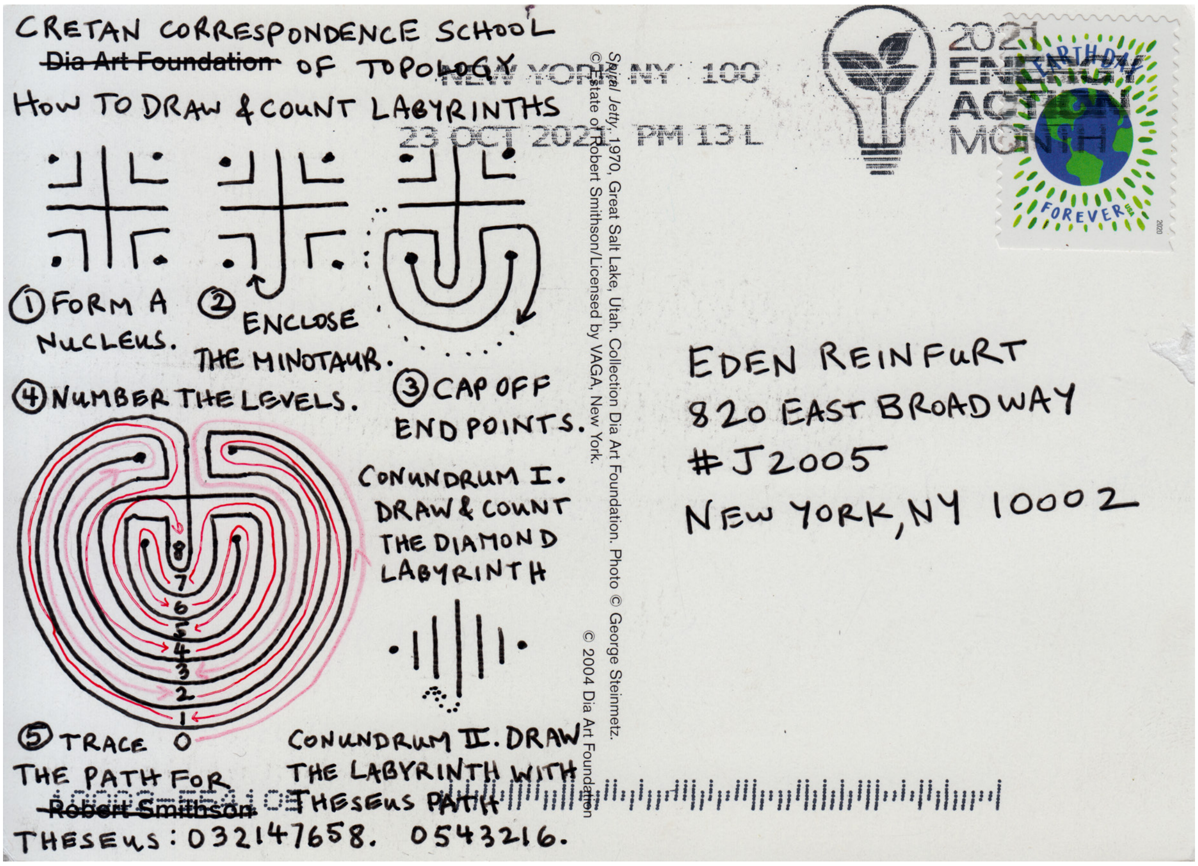
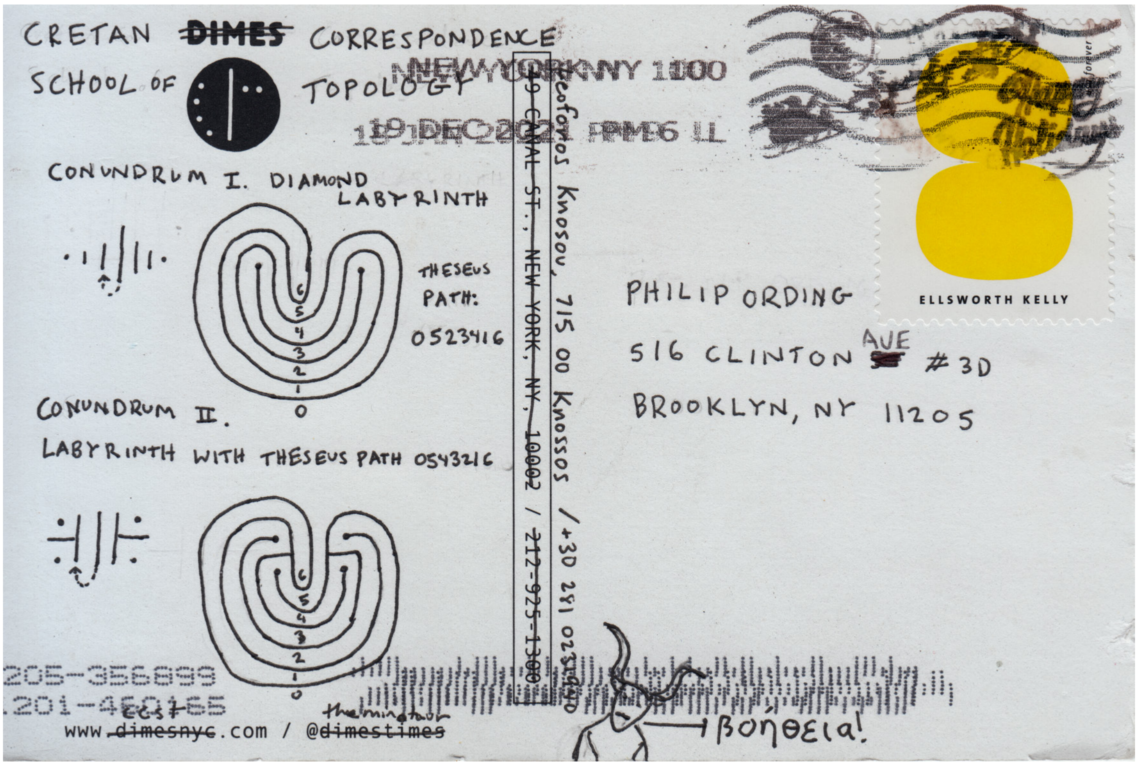


I thought you'd appreciate this postcard since Smithson forgot to build a gift shop on his jetty. His friend Richard said the original design featured just a single hook-like bend out in the lake until someone with gestalty imagination asked: do you intend to build a 7,000-ton question mark? So he went spiral. Were the rocks all salt crystalled when you went? Were you tempted to pocket a souvenir? I don't know why I love that alternating boundary of rock/salt/lake/salt/rock/salt/lake/salt so much. Maybe it activates that part of our animal brain drawn to salt licks.

Or it's just that part of my over-schooled math brain that associates it with the Lakes of Wada — a topological conundrum par excellence — consisting of three simply connected bounded regions of the plane that share a single boundary. In Alexandroff's book (the slim black one with the stained-glass barycentric subdivision on the cover, I think your dad has it) there's a mysterious figure of the Lakes with the caption "Imagine an island in the sea and on it a cold and a warm lake. The following work program is to be carried out on the island. In the course of the first hour canals are to be dug, one from the sea, one from the warm lake, one from the cold lake, in such a way that neither salt and fresh nor warm and cold water come into contact with one another, and so that at the end of the hour every point of land is at a distance of less than one kilometer from each kind of water (i.e. salt, cold, and warm)." The work gets considerably more laborious, as you might expect, in the second hour.

The Cretan labors that I volunteered you for are hopefully less maddening. Or at least interesting enough to elicit conundrums to carry out on your own terms. Like, what happens when I cap a nucleus off willy-nilly?

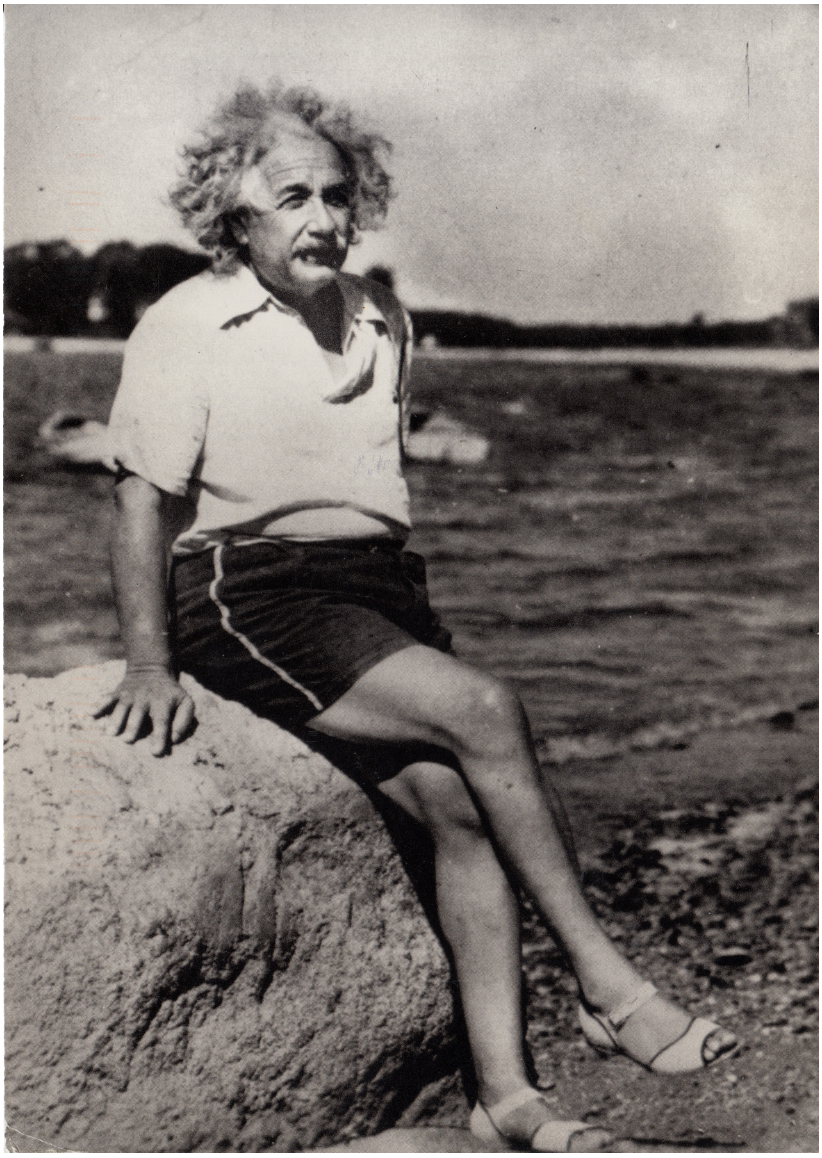
– Philip

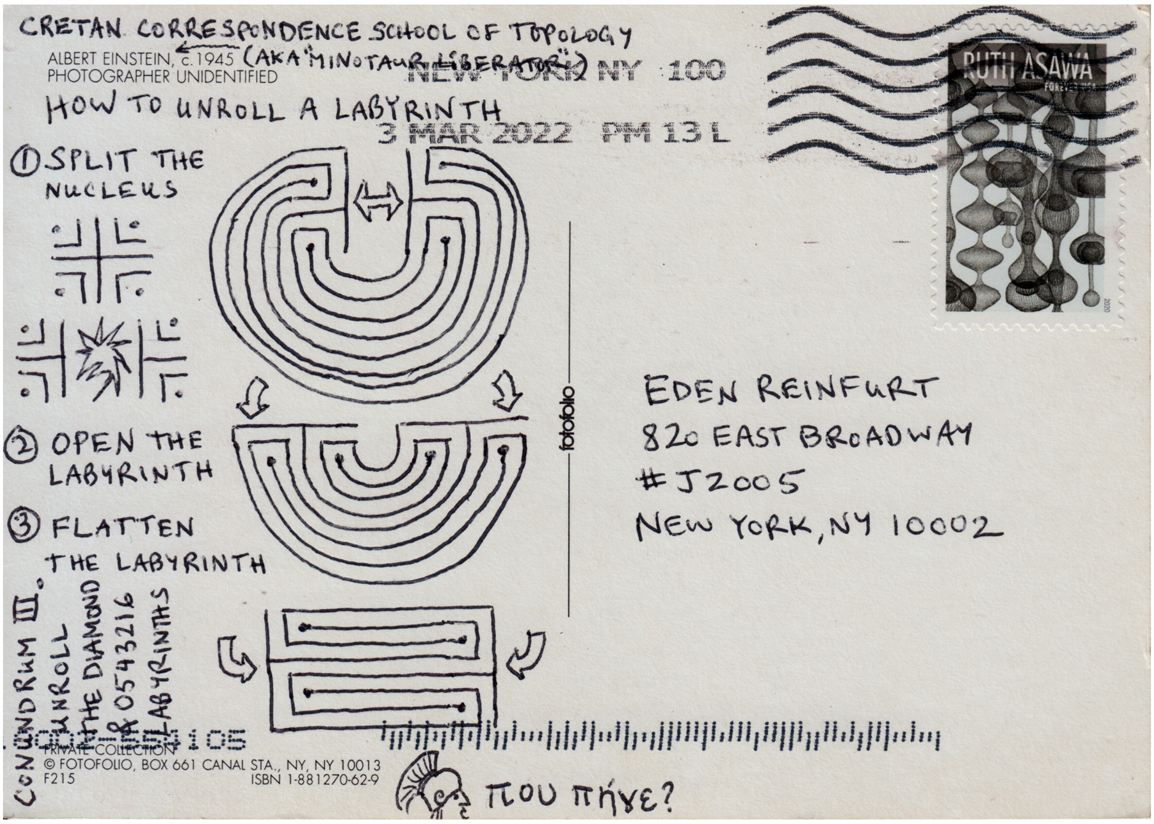


For Conundrum I — the diamond labyrinth — I started very simply, sketching the nucleus and trying to connect the lines together in different ways. I realized that each line of the nucleus had to connect to something else and couldn't branch. Only the two dots at either end of the nucleus could be left as endpoints. But what really solved it for me was figuring out that lines connecting parts of the nucleus would form concentric arcs around the center. From there, I connected all the lines together, starting from the center. Then I labeled the Theseus Path.

Conundrum II was harder. I began by writing out the numbers 6 to 0 vertically, and then tried to draw lines in between that connected them in the order of the Theseus Path. Because it goes in descending order from 5 to 1, I had to make a snaking shape, which then curved around from 1 to go to 6. I took the sides of this snake and placed them around the 6 to form the nucleus. I added dots to form the endpoints, and there ended up being four of them.

– Eden



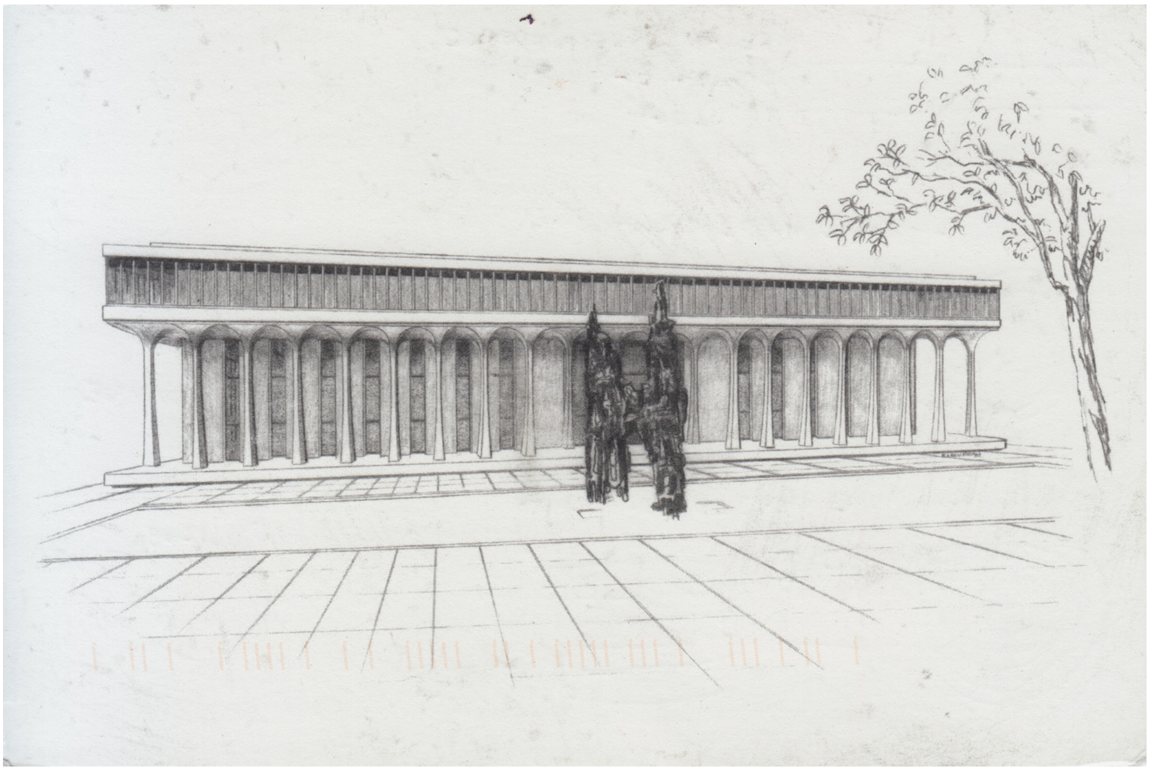


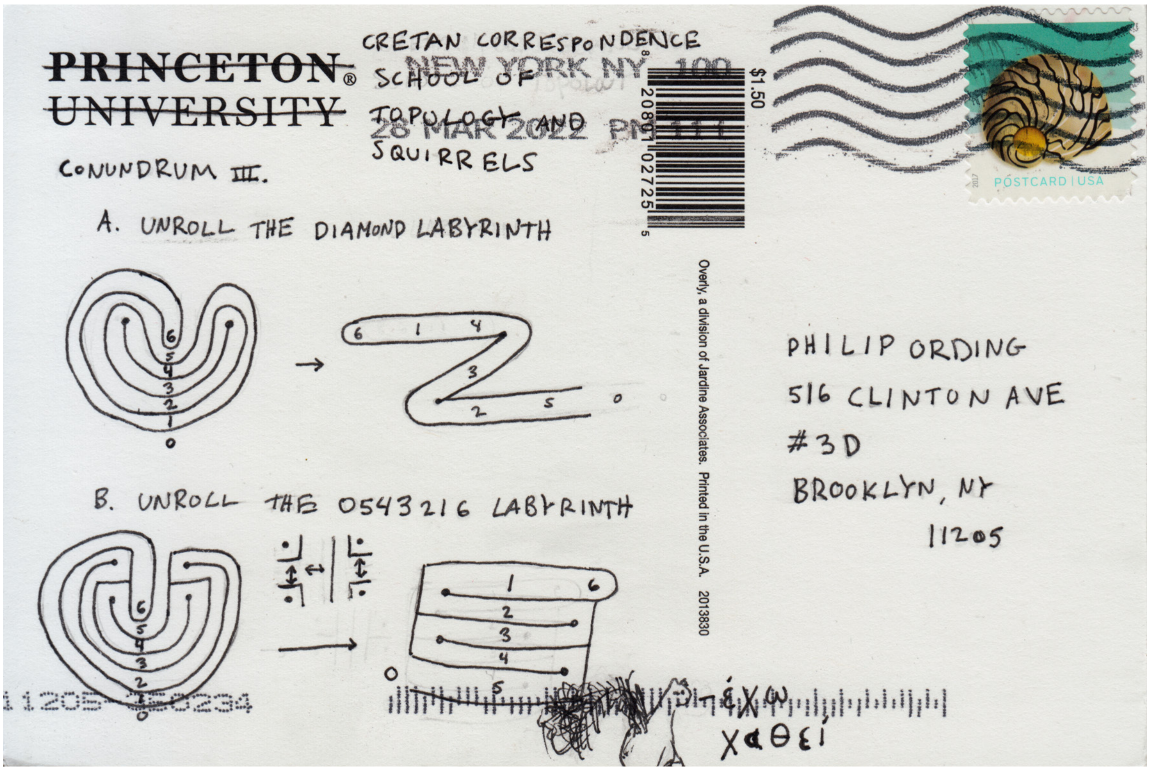
I learned how to draw the Cretan labyrinth from Tony Phillips. That makes it sound like we hang out in bars doodling on napkins. Not quite so, but I did have coffee with the emeritus professor once — turns out he's my biologist colleague Ceci Toro's dad — when he came to campus to talk about mathematical transformations and Bach cannons. As for Professor Phillips, he learned the secret to draw the maze from Jean-Louis Bourgeois, the son of artist Louise Bourgeois and art historian Robert Goldwater. Phillips is also the son of an art historian and likely knew not only the story of Theseus vanquishing the Minotaur and escaping the Cretan labyrinth with Ariadne's thread, but also that the design is older than Methuselah. What seems to have really piqued his mathematical interest, however, were two subsequent encounters.

The first was with a very different — but topologically equivalent — labyrinth on a Native American basket at a friend's home in Arizona. As he explains in his mazes-to-math website chronology, the "basket design is traditional with the To'ono-Otum (formerly Papago) ... living now in northern Arizona, and that the twists and turns of the path through the maze represent events and trials in the life of the hero Iitoi shown at the entrance." And the second was a medieval Hebrew manuscript illustrated with a depiction of the seven walls of Jericho winding concentrically like — but only superficially so — the Cretan labyrinth. What I love about this image is that in place of a hero and heroine's thread, the maze is traced by the reader reading the words of a Psalm. Where did Phillips find this? "Visiting my parents in Florida, and leafing through a Sotheby's catalog."

I trust the fellow illustrated on this postcard in the peep-toes needs no introduction. Less well known among his achievements, especially compared to his own nuclear work, is a brief paper he read before the Prussian Academy entitled "The cause of the formation of meanders in the courses of rivers and of the so-called Baer's Law."

– Philip

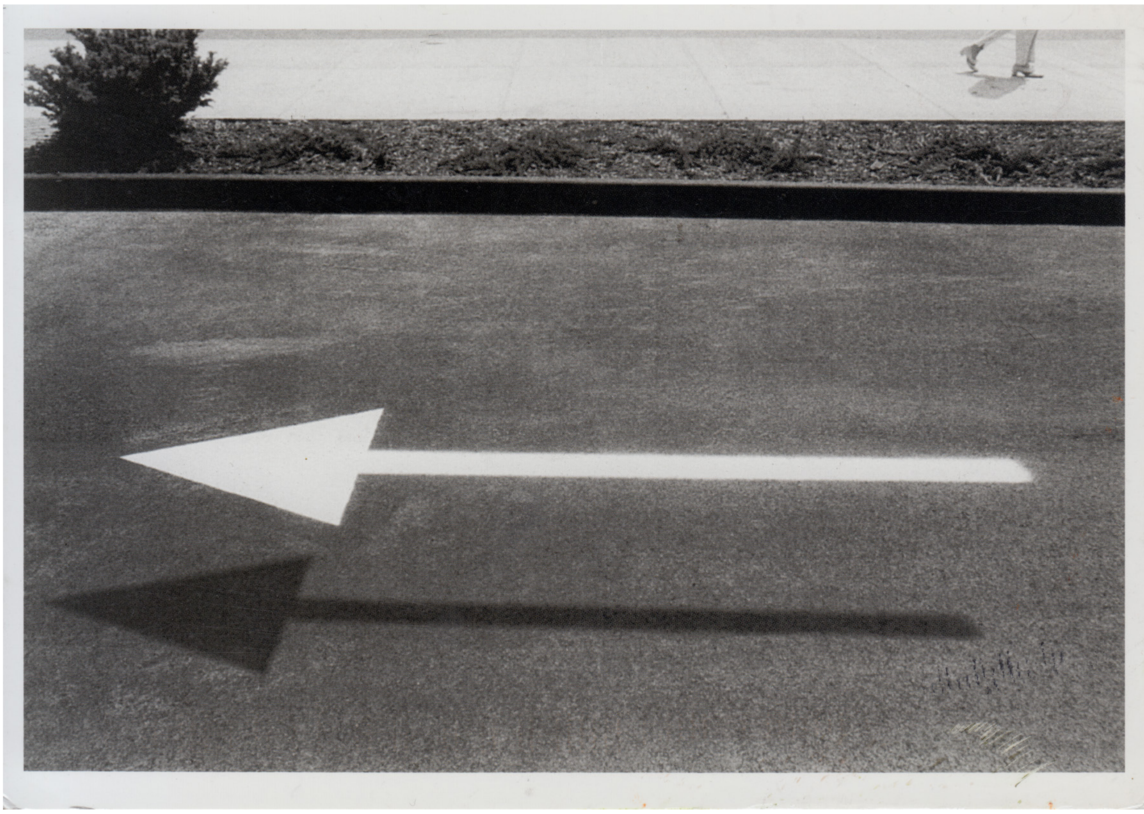


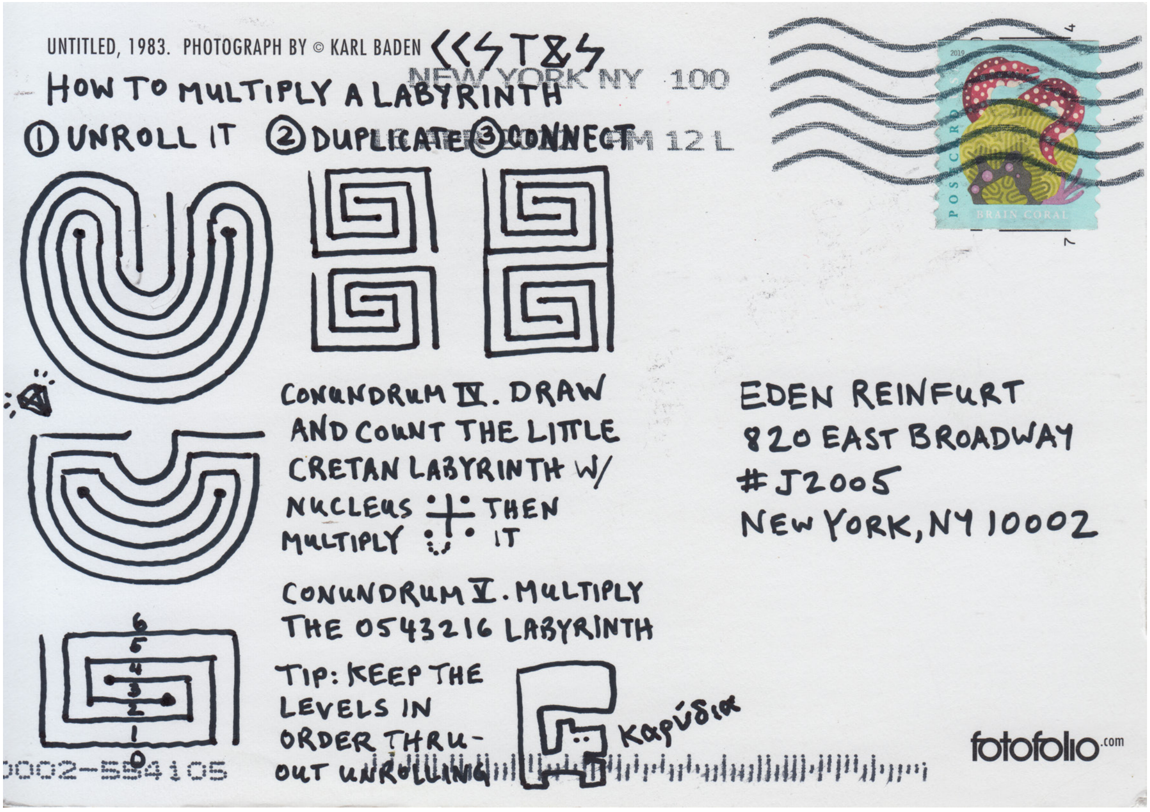


I didn't do Conundrum III quite correctly. For the diamond labyrinth, I split it along almost all of the lines, turning it into a snake. To do that, I imagined that I was taking the labyrinth and pulling it apart. I realize now that I should have cut only the very center, and then unrolled part of it (see the next postcard).

In the 0543216 labyrinth conundrum, I split the nucleus in the way shown in the diagram, breaking apart most of its lines. In hindsight, I should not have split the line connecting the 6 to the left side of the nucleus. Unrolling the labyrinth after I split the nucleus was simple, and I ended up with something almost correct. The only change I should have made was to continue the 6 down the right side of the unrolled labyrinth, and place the 0 at the top of the labyrinth.

—Eden





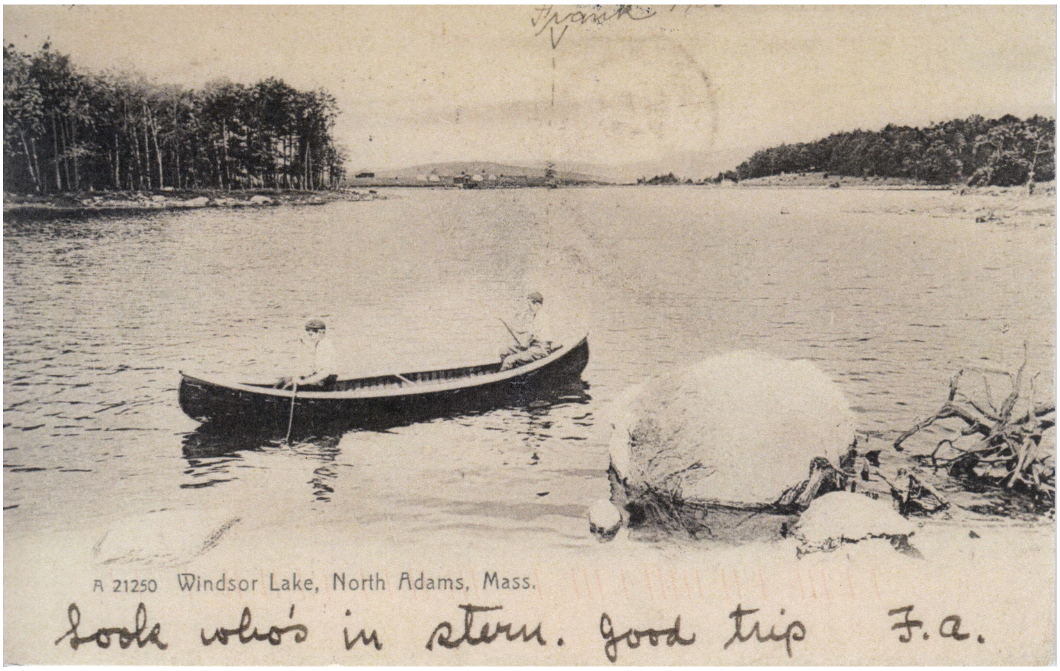
All three mazes that Phillips encountered that fateful winter 40 years ago share three key mathematical properties. Each maze is a transit = no forks in the path from outside to middle. It is alternating = each change of level coincides with a change in direction. And it is simple = the path traverses each concentric level exactly once. These properties allow a complete classification of simple, alternating, transit mazes in terms of the sequence of levels traversed through the labyrinth. (The rightful term for that sequence, by the way, is not Theseus path but Ariadne's thread!) Well, I should qualify: the classification can be done in principle.

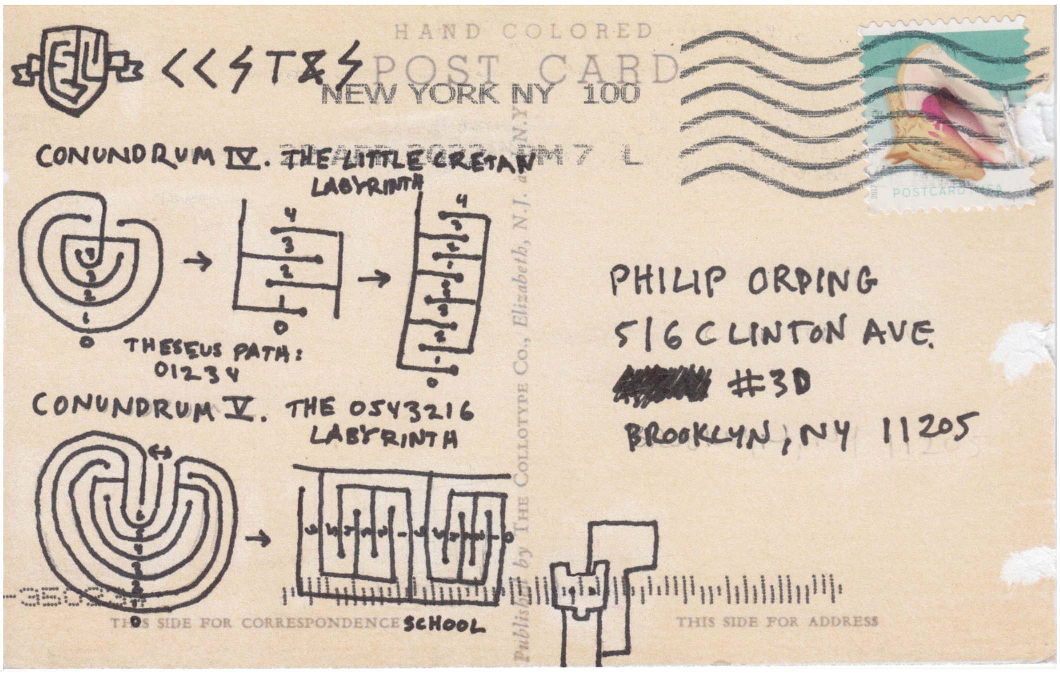
In reality, it turns out to be damn complicated business to enumerate the s.a.t. mazes for an arbitrary number of levels. The count turns out to be related to the number of ways to fold a strip of stamps, at least back when they were perforated and lick-able. Did you ever lick a stamp? I can't say I recommend it. What I can recommend, if you're still in the conundrum mood, is to try figuring out how many s.a.t. mazes have six levels, like the diamond labyrinth. Or, and this is no small feat, what properties a sequence of numbers must satisfy to be Ariadne's thread for some labyrinth.

It's one of those things — like most of math — that I wish I'd first encountered as art. Preferably with friends.

But enough about the unbearable isolation of math, I should tell you how I came to enroll you in the CCST. During the months of pandemic isolation, my friend Liam sent me a postcard from Vancouver with a picture of two beautiful, colorful knots drawn on his office chalkboard. I guess it could have been in his living room — you can't be sure with mathematicians. On the reverse, he presented a new theorem, which the diagrams illustrate, complete with proof and citations. A math paper in miniature, the first in a series he's calling Mathematical Research Postcards. It made me want to try.

– Philip


A 21250 Windsor Lake, North Adams, Mass.
Look who's in stern. Good trip F.A.



For Conundrum IV, I first drew the little Cretan labyrinth, connecting the lines and dots of the nucleus. Like the first conundrum, I started from the very center and worked my way outwards. The Theseus Path ended up being 01234. To unroll it, I split the nucleus at the center vertical line and imagined pulling it apart to the sides. I made sure to keep the levels in order in the unrolled version. Then I replicated the unrolled labyrinth and connected them vertically.

I had already unrolled the 0543216 labyrinth, although incorrectly, so I had some idea of how to start to solve this conundrum. First, I split the nucleus so that the center wasn't closed off. Then I unrolled it, making sure that the levels were in order. One thing I realized was that the ends—the 6 and the 0 levels—had to be at opposite ends of the unrolled labyrinth so that it could be multiplied. To do that, I made those levels curve around the outside of the labyrinth. That let me duplicate it and connect them. But there was not a lot of space on this postcard so I had to draw the labyrinth on its side.

I enjoyed solving these problems, even when I didn't get them exactly right, and I learned a lot in the process. The format of the assignments, as postcards, made it much more interesting. It limited the space I could use to convey how I solved the problem, and made this much more of a visual process. It was not a fast process either. Whenever we received a postcard, it would sit around for a while, before we finally got to working on it. Then my dad and I would look at it after dinner and try to solve it, sketching it out on paper. It was a lot of fun. And there were squirrels.

—Eden